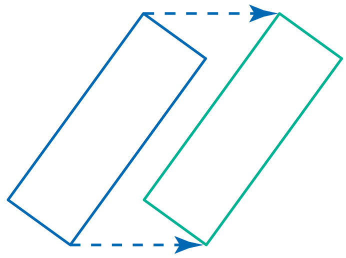
LESSON 1

Transformations and Congruence

Rigid Motions

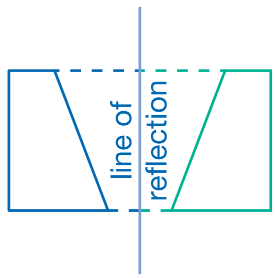
UNDERSTAND A **transformation** involves moving all of the points that make up a geometric figure according to the same rule. The original figure is called the **preimage**, and the transformed figure is called the **image**. Transformations that change the position of a figure without changing its shape or size are known as **rigid motions**.

A translation is a rigid motion that slides a figure to a new location. In a translation, all points are moved the same distance in the same direction. A figure may be translated any distance and in any direction.

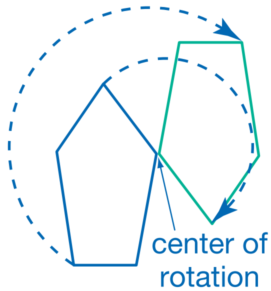


In the example on the right, the blue rectangle is the preimage and the green rectangle is the image. The dashed-line arrows illustrate how two of the points move during the translation. Notice that the arrows are the same length because the points move the same distance. The arrows are also parallel, because the points move in the same direction.

A reflection is a rigid motion that flips a figure over a line, called the line of reflection. After a reflection, each point on the image lies the same distance from the line of reflection as its corresponding point on the preimage, but on the opposite side. A reflection over a line produces a mirror image of the original preimage.

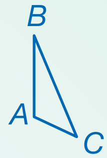


A rotation is a rigid motion that turns a figure about a point, called the center of rotation. A rotation moves the points in a figure along curved paths around the center of rotation. Each point is moved the same number of degrees around the center. After a rotation, each point on the image lies the same distance from the center of rotation as its corresponding point on the preimage.



Connect

Sketch the images formed when $\triangle ABC$ is transformed in three different ways. Rotate $\triangle ABC$ 90° clockwise around point C, translate $\triangle ABC$ to the right, and reflect $\triangle ABC$ across a vertical line.

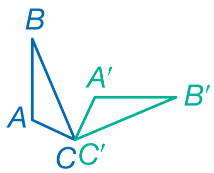


1

Perform the rotation.

Imagine placing one finger on vertex C and using another finger to turn the rest of the triangle about that point. The shape that results from this is the image.

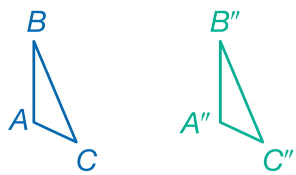
Name the new vertices using the ′ mark. The vertex A' is read as *A prime*.



2

Perform the translation.

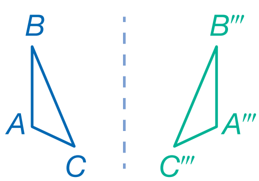
Imagine pushing on the figure and sliding it to the right. This action is a translation. Name the vertices of this figure using the ″ mark, to differentiate it from the image produced in Step 1. The vertex A'' is read as *A double prime*.



3

Perform the reflection.

Imagine picking the triangle up and flipping it over, like a pancake. Or, think of placing a mirror to the right of the figure and looking at its image in the mirror. The triangle you would observe in either case would be the result of a reflection. The vertex A''' is read as *A triple prime*.



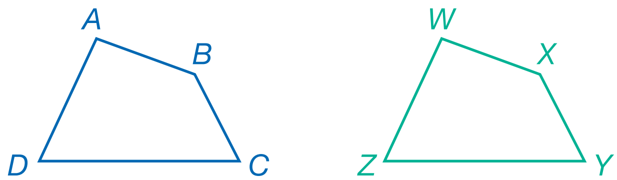
DISCUSS

Compare the three images formed. How are they similar? How do they differ?

Congruence and Coordinates

UNDERSTAND A polygon is a closed figure made up of straight-line sides connected by vertices. The side between two vertices is the **line segment** that has those vertices as endpoints. The length of a side is equal to the distance between its two endpoints, or between the vertices. Whenever two line segments share a common endpoint, they form an **angle**.

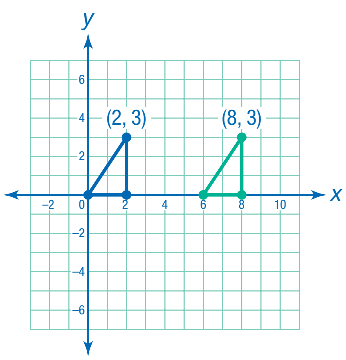
Two polygons are **congruent** if all of the following are true: they have the same number of sides and angles, corresponding sides have the same length, and corresponding angles have the same measure. Sides or angles are corresponding if they are in the same location in a figure. For example, quadrilaterals *ABCD* and *WXYZ* are congruent. Angle *W* corresponds to angle *A*, so these two angles are congruent, or have the same measure. Side $\overline{XY}$ corresponds to side $\overline{BC}$ so these two sides are congruent, or have the same length.



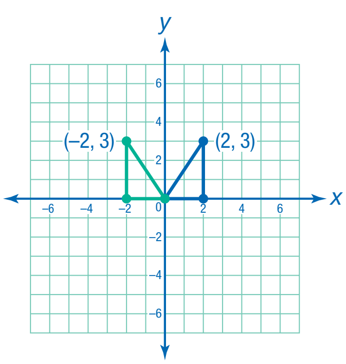
UNDERSTAND Rigid motions preserve angle measures and lengths of line segments. This means that when a rigid motion is performed on a figure, the corresponding sides and angles of the image and the preimage are congruent. In other words, after a rigid motion, the image is congruent to the preimage, though often in a different location and sometimes having a different orientation, meaning it is turned or flipped.

You can prove two figures are congruent by finding a rigid motion (or series of rigid motions) that produces one figure from the other. When the figures graphed on a coordinate plane undergo rigid motions, the coordinates of the points that make up the figure change in predictable ways.

On the first coordinate plane on the right, the blue triangle was translated 6 units to the right. The vertex (2, 3) on the preimage maps to the point (2 + 6, 3), or (8, 3), on the green image. For each point on the preimage, the corresponding point on the image can be found by adding 6 to the *x*-coordinate. By showing that each point of the image is a result of the same transformation of a point from the preimage, you can prove that the triangles are congruent.

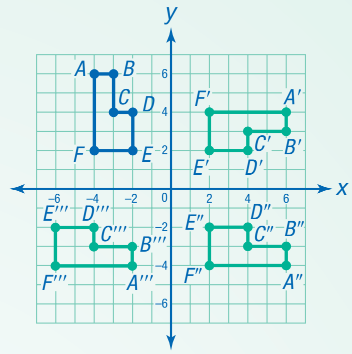


On the second coordinate plane, the blue triangle was reflected over the *y*-axis. The vertex (2, 3) on the preimage maps to the point (−2, 3) on the image. For each point on the preimage, the corresponding point on the image can be found by changing the sign of the *x*-coordinate.



Connect

Figure *ABCDEF* was put through a series of transformations. Identify each transformation.



1

Identify the transformation from *ABCDEF* to $A'B'C'D'E'F'$.

The figure has been turned on its side. Vertex *A* is in the upper left corner of *ABCDEF*, but A' is in the upper right corner of $A'B'C'D'E'F'$. However, the vertex names are in the same order when read clockwise. The figure appears to have been rotated. Notice that each point on $A'B'C'D'E'F'$ is the same distance from the origin as its corresponding point on *ABCDEF*. Figure *ABCDEF* was rotated around the origin.

2

Identify the transformation from $A'B'C'D'E'F'$ to $A''B''C''D''E''F''$.

Figure $A''B''C''D''E''F''$ is a mirror image of figure $A'B'C'D'E'F'$. Vertex A' is in the upper right corner of $A'B'C'D'E'F'$, but A'' is in the lower right corner of $A''B''C''D''E''F''$. The vertex names are in a different order when read clockwise. The figure appears to have been reflected. Each point on $A''B''C''D''E''F''$ is the same distance from the *x*-axis as its corresponding point on $A'B'C'D'E'F'$. Figure $A'B'C'D'E'F'$ was reflected across the *x*-axis.

3

Identify the transformation from $A''B''C''D''E''F''$ to $A'''B'''C'''D'''E'''F'''$.

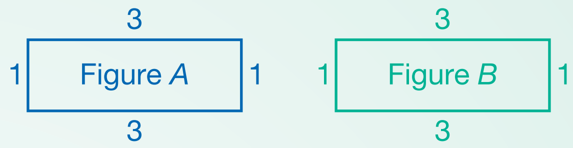
Figure $A'''B'''C'''D'''E'''F'''$ is oriented the same way as figure $A''B''C''D''E''F''$, but it was shifted to a different location on the graph. Vertex A'' is in the lower right corner of $A''B''C''D''E''F''$, and A''' is in the lower right corner of $A'''B'''C'''D'''E'''F'''$.

Since the orientation did not change, figure $A'''B'''C'''D'''E'''F'''$ is the result of a translation, 8 units to the left.

DISCUSS

Compare the figures. Did the size or shape of the figure change as it was transformed? How could you confirm this?

EXAMPLE A Identify transformations that could be applied to figure *A* to form figure *B*.



1 Compare the size and shape of the image and the preimage.

Figures *A* and *B* are rectangles with the same height and width, so they have the same size and shape. This means that figure *B* could be the result of one or more rigid transformations of figure *A*.

2 Determine if figure *A* can form figure *B* through a translation.

The two figures have the same orientation, but figure *B* is about 1 unit to the right of figure *A*. The upper left corner of figure *B* is 4 units to the right of the upper left corner of figure *A*. In fact, this is true about all corresponding points on the two figures.

▶ A translation 4 units to the right will transform figure *A* into figure *B*.

3 Determine if figure *A* can form figure *B* through a reflection.

Figure *B* is a mirror image of figure *A*. A vertical line drawn half-way between the figures could serve as a line of reflection.

▶ A horizontal reflection will transform figure *A* into figure *B*.

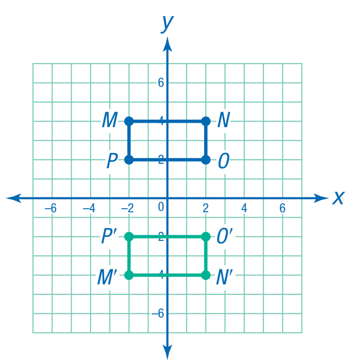
4 Determine if figure *A* can form figure *B* through a rotation.

If you were to turn this page upside down, the figures would look the same. Turning the figures upside down is a way of rotating them. The center of rotation lies halfway between the figures.

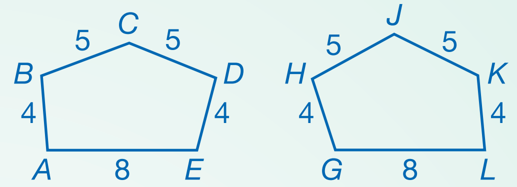
▶ A half-turn rotation, or a rotation of 180°, will transform figure *A* into figure *B*.

DISCUSS

What transformations can be applied to *MNOP* to form *M′N′O′P′*?



EXAMPLE B Determine if the two pentagons are congruent.



1 Compare the side lengths.

Beginning at vertices A and G and moving clockwise, we can see that corresponding sides are congruent.

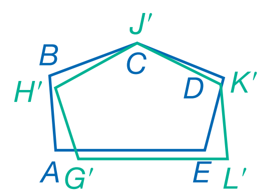
$AB = GH = 4$ $BC = HJ = 5$

$CD = JK = 5$ $DE = KL = 4$

$AE = GL = 8$

2 Align one pair of vertices.

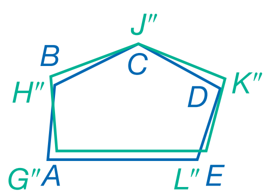
Vertices J and C both lie opposite to sides measuring 8 units. Translate pentagon $GHJKL$ left so that vertex J lies on top of vertex C.



The image produced by the translation does not appear to cover $ABCDE$.

3 Try another rigid motion.

Notice that $BC = HJ = CD = JK = 5$ and $AB = GH = DE = KL = 4$. This means that the sides closest to each other will still be congruent after a horizontal reflection of one of the figures. Try reflecting $G'H'J'K'L'$ across a vertical line through J' to see if this causes the sides to line up.



The image produced by the reflection does not appear to cover $ABCDE$.

▶ Rigid motions will not carry $GHJKL$ onto $ABCDE$, so the figures are not congruent.

MODEL

Suppose you were given two other pentagons. If all of the corresponding angle measures were congruent, can the pentagons be proven congruent? Create a sketch to support your answer.

Practice

Write an appropriate word or phrase in each blank.

1. A(n) ________________ is the part of a line that falls between two points, called endpoints.

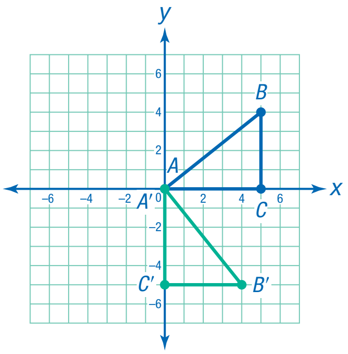
2. A(n) ________________ is formed by two line segments or rays that have a common endpoint

3. A(n) ________________ is a slide of a figure to a new location.

4. Two figures are congruent if their corresponding sides have equal ________________ and their corresponding ________________ have equal measure.

Identify a transformation that could be applied to $\triangle ABC$ to form $\triangle A'B'C'$.

5.



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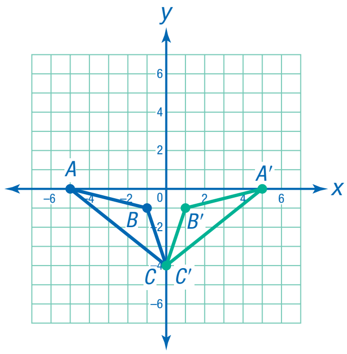
6.



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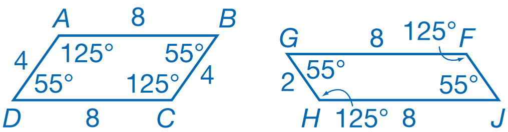
REMEMBER A reflection is a flip, a rotation is a turn, and a translation is a slide.

For questions 7 and 8, determine if each pair of figures are congruent. If so, describe rigid motions that would carry one figure on to the other.

7.



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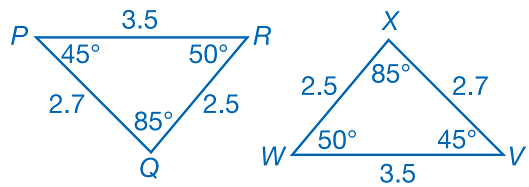
8.



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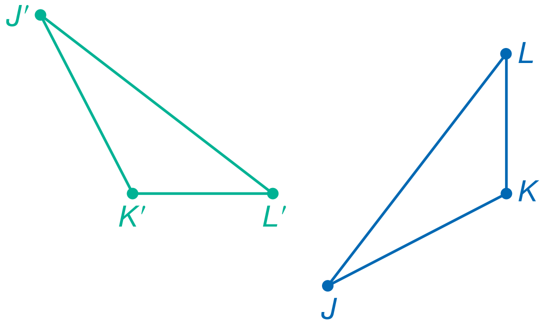
REMEMBER Congruent figures have all corresponding sides and angles congruent.

Choose the best answer.

9. Which rigid motions could be used to transform $\triangle JKL$ into $\triangle J'K'L'$?



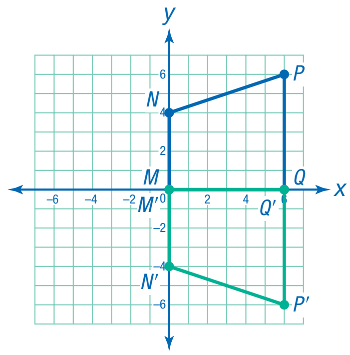
A. translation of $\triangle JKL$ up and to the left

B. translation of $\triangle JKL$ down and to the right

C. 90° clockwise rotation of $\triangle JKL$ about point K followed by a translation to the left

D. 90° counterclockwise rotation of $\triangle JKL$ about point K followed by a translation to the right

10. Across which line was trapezoid $MNPQ$ reflected to form trapezoid $M'N'P'Q'$?



A. the x-axis

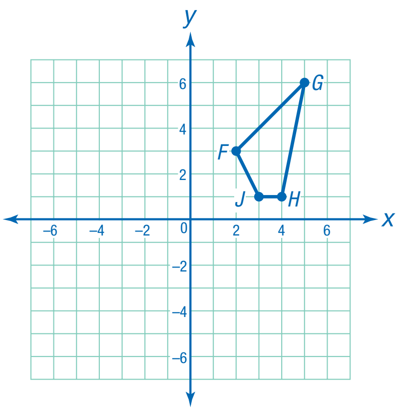
B. the y-axis

C. the line $y = x$

D. the line $y = -x$

Solve.

11. **ROTATE** Rotate quadrilateral $FGHJ$ 180° about the origin to form quadrilateral $F'G'H'J'$. Draw the image $F'G'H'J'$ on the coordinate plane.



12. **PREDICT** Mei-Lin uses a computer drawing program to draw a polygon. She then uses the program to copy the polygon, to rotate the copy 90°, and then to flip it horizontally. How will her original drawing compare to the final image? Will the length of the sides be the same in both drawings? Will the angle measures be the same? Explain how you know.